Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Higher level
Paper 3

20 May 2026

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

1 hour 15 minutes

Instructions to students

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[55 marks]**.

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9 pages

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Answer **both** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 26]

This question explores statistical tests to decide whether the use of mobile phones (cell phones) by students in schools affects their performance.

In a number of schools worldwide, the use of mobile phones has been restricted or not allowed.

Ling, a student, decides to run her own investigation. She performs several tests, all at the 5% significance level, to determine whether the use of mobile phones in schools has a negative effect on achievement for students aged between 12 and 15 years.

Initially, Ling compares two schools. School A has not been allowed to use mobile phones, and school B has no restrictions. A quiz is completed by all students at these schools. Ling takes a random sample of the results of this quiz from each school.

Ling's results are summarized in **Table 1**.

Table 1

School	Sample size, n	Mean, $\bar{x}$	Standard deviation, s_n
A	63	56	12.2
B	39	51	14.5

(a) (i) State an appropriate test to determine whether there is evidence that the mean result of the quiz of the students in school B is lower than in school A. [1]

(ii) State **one** assumption that is made in order to carry out the test from part (a)(i). [1]

(iii) State the null and alternative hypotheses. [2]

(iv) Find the p-value. [3]

(v) State the conclusion for the test in context. Give a reason for your answer. [2]

(b) State a reason why this statistical test should not be considered strong enough evidence to decide upon the effect of mobile phone use in schools. [1]

(This question continues on the following page)

(Question 1 continued)

To gain a better understanding of the effect of mobile phone use, Ling collects marks, $m \in \mathbb{Z}$, from a national examination taken by students from different schools. Some of the schools had a complete ban on mobile phones, others allowed only a restricted use, and the remaining schools had unrestricted use.

The results are given in **Table 2**.

Table 2

	School's mobile phone policy			
Marks, $m \in \mathbb{Z}$	**Complete ban**	**Restricted use**	**Unrestricted use**	**Total**
$\mathbf{1 \leq m \leq 19}$	4	7	6	17
$\mathbf{20 \leq m \leq 34}$	8	12	10	30
$\mathbf{35 \leq m \leq 49}$	27	27	23	77
$\mathbf{50 \leq m \leq 64}$	48	44	27	119
$\mathbf{65 \leq m \leq 79}$	53	43	16	112
$\mathbf{80 \leq m \leq 100}$	20	17	8	45
Total	160	150	90	400

Ling calculated the estimated means for the students with a complete ban and with restricted use to be 60.9 and 57.7 respectively.

(c) Find the estimated mean for the students with unrestricted use. [3]

Based on the estimated means, Ling believes that achievement depends on the use of mobile phones and decides to perform a χ^2 test for independence on the data from **Table 2**.

In order to perform the test, Ling combines two rows in **Table 2**.

(d) (i) State, with a justification, which two rows Ling combines. [2]

(ii) Perform the test to determine whether there is sufficient evidence to support Ling's belief. [4]

(This question continues on the following page)

(Question 1 continued)

For her final investigation, Ling decides to take a sample of 60 students from schools with a complete ban on mobile phones. She tests whether their mean mark on a different national examination is higher than the national mean mark of 64.8.

The population standard deviation is 15.3 marks. It is assumed that the standard deviation for students with a complete ban on mobile phones will be the same. The marks in this different national examination are normally distributed.

(e) State the probability that Ling makes a Type I error, given that the null hypothesis is true. [1]

Based on a nationwide investigation, it is expected that for students with a complete ban of mobile phones the mean mark is 68.3. You may assume that this is the case.

(f) (i) Find the critical region for the test. [3]

(ii) Find the probability that Ling makes a Type II error. [1]

One option to reduce the probability of making a Type II error is to increase the sample size.

(g) (i) Suggest a different option to reduce the probability of making a Type II error. [1]

(ii) Describe a disadvantage of your answer to part (g)(i). [1]

2. [Maximum mark: 29]

This question explores how operations on complex numbers can represent the transformation of points in order to create animated images.

In an animation, a series of images, called frames, are produced such that showing the frames one after another creates the illusion of a moving object.

Movements are created using transformations, such as rotations, enlargements and displacements with constant velocity along a line or a curve.

An object is animated by applying the same transformation to all its points.

A point with coordinates (x, y) can be represented by the complex number $z = x + y\mathrm{i}$.

(a) Let z be multiplied by i.

(i) Find $z\mathrm{i}$ in Cartesian form. [1]

(ii) Describe the rotation that corresponds to multiplying by i. [2]

(b) Describe the transformation that corresponds to adding $a + b\mathrm{i}$ to z, where $a, b \in \mathbb{R}$. [1]

(c) Write down the operation that corresponds to an enlargement with centre $(0, 0)$ and scale factor $k \in \mathbb{R}^+$. [1]

Consider the functions defined by $f(z) = 2\mathrm{i}z$ and $g(z) = z + 2 + \mathrm{i}$.

(d) (i) Write down an expression for $g^{-1}(z)$. [1]

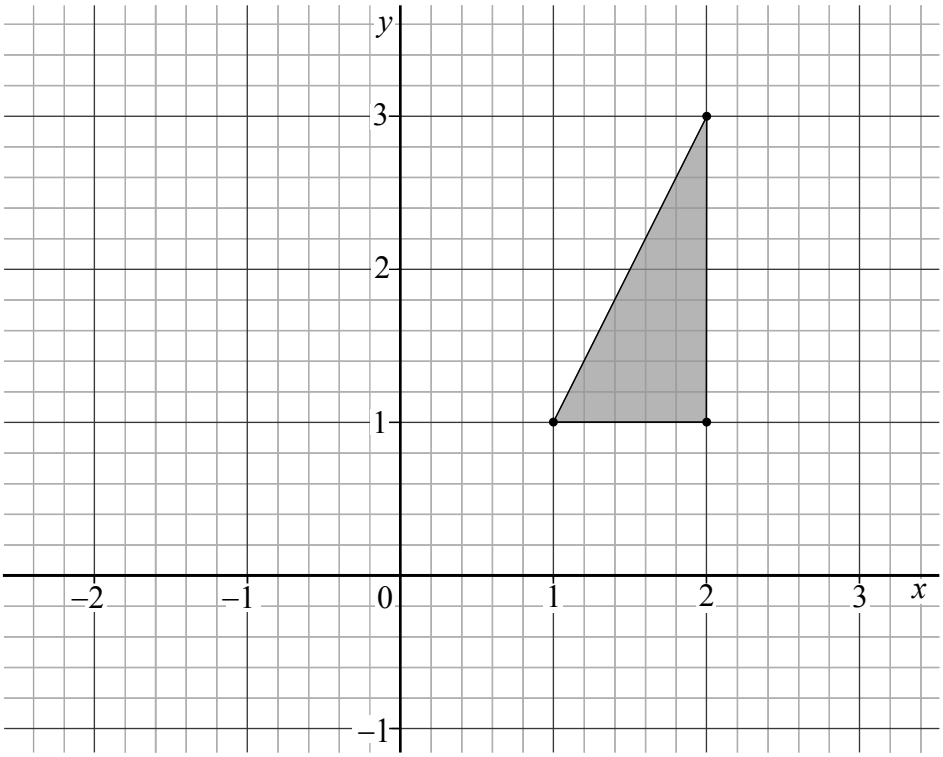
(ii) Given that $h(z) = (g \circ f \circ g^{-1})(z)$, show that $h(z) = 2\mathrm{i}z + 4 - 3\mathrm{i}$. [2]

(This question continues on the following page)

(Question 2 continued)

Triangle ABC is defined by points A, B and C, represented by the complex numbers $z_A = 1 + \mathrm{i}$, $z_B = 2 + \mathrm{i}$ and $z_C = 2 + 3\mathrm{i}$ respectively. **Diagram 1** shows triangle ABC.

Diagram 1



Reminder: any work written in this question booklet will not be marked

(e) (i) Find the coordinates of points A', B' and C', represented by the complex numbers $h(z_A)$, $h(z_B)$ and $h(z_C)$ respectively. [3]

(ii) Hence, by sketching triangles ABC and $\mathrm{A'B'C'}$, or otherwise, describe $h(z)$ as the composition of a rotation and an enlargement. [3]

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(Question 2 continued)

To animate a point represented by $z \in \mathbb{C}$, functions of the type $F(z, t)$ are created, such that $F(z, t)$ represents the image of the point at t seconds since the start of the animation.

For example, the function $R_{30}(z, t) = z \times \left(\cos(30t°) + \mathrm{i}\sin(30t°)\right)$ and results in an anticlockwise rotation of $30°$ per second about $(0, 0)$.

(f) Write down an expression for the transformation $R_\theta(z, t)$ resulting in an anticlockwise rotation of θ degrees per second about $(0, 0)$. [1]

(g) Find an expression for $T(z, t)$, which results in a displacement with a constant velocity of $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$ per second. [2]

To create an object which is increasing or decreasing in size, a function of the type $E_k(z, t) = \left(1 + (k - 1)t\right)z$, where k is a real constant, can be used to obtain the image of the object. The function is applied to the complex representations of all points of the object.

$E_3(z, t)$ is used to obtain the image of a circle, C, with radius 1 unit, after t seconds.

(h) Find the radius of the image of C after

(i) 1 second. [2]

(ii) 2 seconds. [1]

(i) Explain why applying $E_k(z, t)$ to an object results in the object increasing or decreasing in size at a constant rate. [2]

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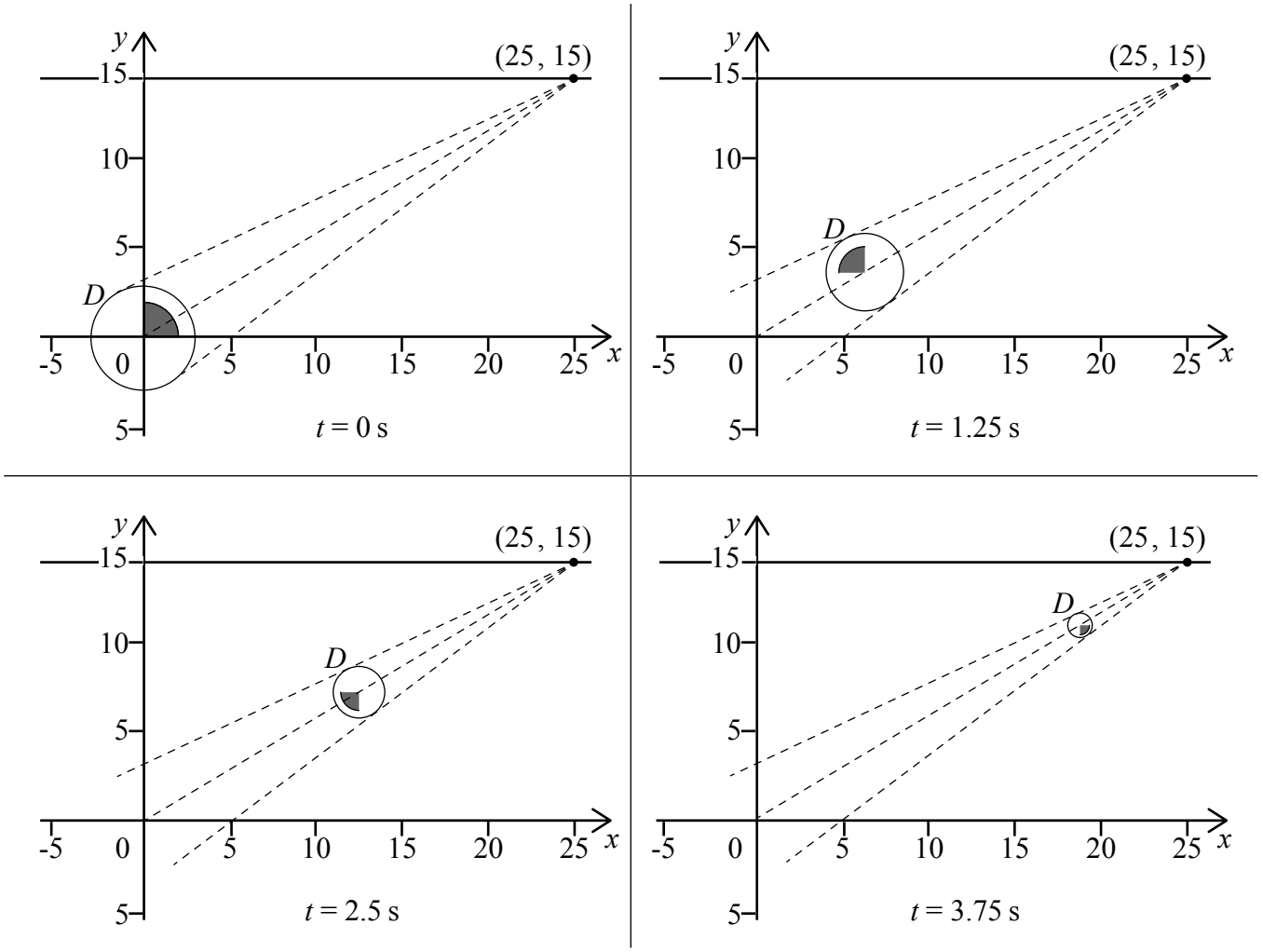
(Question 2 continued)

Matt wants to create an animation showing a rotating circular disc, D, with a portion shaded in, disappearing at the horizon. Four frames from Matt's animation are shown.

diagram not to scale



At $t = 0$, the disc has its centre at $(0,\ 0)$ and a radius of 3 units. After 5 seconds, the disc "disappears" at point $(25,\ 15)$ when its radius is reduced to 0. The disc rotates $360°$ per second anticlockwise about its centre.

Let the function used to create this animation be denoted by $A(z,\ t)$ for $0 \le t \le 5$.

(j) Find an expression for $A(z,\ t)$. [5]

Matt uses 60 frames per second in his animation. His initial frame, at $t = 0\,\text{s}$, is denoted frame zero.

(k) Find the transformation $B_n(z)$ used to obtain the image of the disc in the nth frame. [2]